A STRESS–STRAIN STATE IN CAPILLARY COOLING SYSTEMS WITH LOCAL HEATING

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We have studied the stress–strain state of a transversally isotropic, in a thermal respect, half-space cooled by an opposing-reverse coolant flow. We have analyzed the effect of coolant heating on temperature, deflection, and stresses arising at the surfaces heated by an axisymmetric heat flux.

A cooling system of the mirrors of laser technological units is intended for reducing strains and stress on a heated surface and represents the capillary network with a developed heat-transfer surface. Figure 1 illustrates a capillary cooling system with an opposing-reverse coolant flow. A coolant is fed through one capillary system, arranged perpendicularly to the heated surface, and removed through another. The characteristic hydraulic diameter and thickness of the walls between two neighboring capillaries constitute tenths of a millimeter, and their number in the heated zone amounts to several thousand. Therefore, a classic approach to solving the heat conduction and thermoelasticity problem with a statement of boundary conditions at the capillary walls necessitates excessive computational resourses.

Recently such a cooling system has been replaced in modeling by a porous medium in which the coolant flows. Thermophysical characteristics are specified with the help of presented thermal conductivity and heat-transfer coefficients, as well as elasticity moduli determined experimentally. Among the literature cited, such an approach is employed in [1-3] for describing heat exchange in the cooling systems of the mirrors of laser technological units. Their authors assumed a coolant temperature invariable across thickness. However, it is difficult to practically provide the coolant flow rates sufficient to neglect the effect of the coolant heating on the mirror characteristics. The current work is one of the first to have studied the stress–strain state of the capillary cooling systems with a view of the influence of the coolant heating. The regions of varying filtration rate and volume heat-transfer coefficients, where this influence is inessential, have been identified.

The cooling system is regarded as a transversally isotropic, in thermal respect, porous medium with an isotropy plane normal to the capillary direction. Let T_1 and T_2 be temperatures of the fed and removed coolant. A temperature drop ΔT across a framework wall is of the order

$$\Delta T \sim (T_2 - T_1)\frac{\alpha\delta}{\lambda},$$

where α is the characteristic surface coefficient of heat transfer at the capillary wall, δ is the wall thickness, and λ is the material thermal conductivity.

In the considered cooling systems $\alpha\delta/\lambda \sim 0.01$, therefore, it is possible to neglect the temperature drop across the capillary walls and define the framework temperature by a mean temperature T. The heat-transfer coefficients and the filtration rate are assumed to be constant throughout the cooling system, and the heat flux is taken to be axisymmetric.

The heat-conduction and heat-transfer problem is formulated as follows. In a half-space $z \geq 0$, three functions $T(r, z)$, $T_1(r, z)$, and $T_2(r, z)$ need to be determined, which satisfy the equations

$$\begin{gathered}\frac{\partial^2 T}{\partial z^2} + \frac{\lambda_r}{\lambda_z}\nabla_r^2 T = \alpha_1(T - T_1) + \alpha_2(T - T_2),\\ \frac{\partial T_1}{\partial z} = \alpha_1 k(T_1 - T),\\ \frac{\partial T_2}{\partial z} = -\alpha_2 k(T_2 - T).\end{gathered} \tag{1}$$

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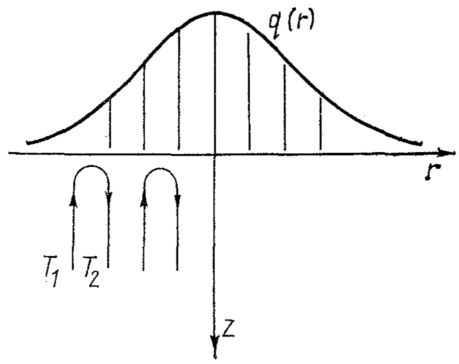

Fig. 1. Diagram of capillary cooling systems.

On the surface z = 0, a heat flux is imposed and the conditions of temperature conjugation for the supplied and removed coolants must be fulfilled

$$-\lambda\frac{\partial T}{\partial z}=q(r),\ T_1(r,\ 0)=T_2(r,\ 0).$$

The functions T, T_1, and T_2 tend to zero at infinity. The Hankel transform with respect to r yields a system of ordinary differential equations for the transforms of unknown functions T^*, $T_1{}^*$, and $T_2{}^*$:

$$\begin{gathered}\frac{d^2}{dz^2}T^*=\alpha_1(T^*-T_1^*)+\alpha_2(T^*-T_2^*)+\xi_1^2T^*,\\ \frac{d}{dz}T_1^*=\alpha_1k(T_1^*-T^*),\\ \frac{d}{dz}T_2^*=-\alpha_2k(T_2^*-T^*).\end{gathered}\tag{2}$$

A characteristic equation for this system is of the fourth order:

$$\begin{gathered}\omega^4-\omega^3k(\alpha_1-\alpha_2)-\omega^2(\alpha_1\alpha_2k^2+\alpha_1+\alpha_2+\xi_1^2)+\\ +\omega k(\alpha_1-\alpha_2)\xi_1^2+\xi_1^2k\alpha_1\alpha_2=0.\end{gathered}\tag{3}$$

Let the free term in Eq. (3) be positive, then the entire left side is positive for $\omega\to\pm\infty$ and $\omega = 0$. When $\omega = \pm\xi_1$, the left side takes negative values $-\xi_1{}^2(\alpha_1 + \alpha_2)$. Therefore, Eq. (3) has four real roots, of which two are positive (ω_1 and ω_2) and two negative (ω_3 and ω_4). Since the intervals of sign reversal are known, the roots are easy to determine numerically.

In some cases, we succeed in writing explicit expressions for the roots.

1. **Thermal insulation of offtakes** ($\alpha_2 = 0$):

$$\begin{gathered}\omega_1=\cos\frac{\varphi}{3}\left[\frac{2}{\sqrt{3}}\sqrt{\frac{k^2\alpha_1^2}{3}+\alpha_1+\xi_1^2}\right]+\frac{k\alpha_1}{3},\\ \omega_{2,3}=\cos\left(\frac{\varphi}{3}\mp\frac{2\pi}{3}\right)\left[\frac{2}{\sqrt{3}}\sqrt{\frac{k^2\alpha_1^2}{3}+\alpha_1+\xi_1^2}\right]+\frac{k\alpha_1}{3},\\ \omega_4=0,\end{gathered}\tag{4}$$

where

$$\cos\varphi=\frac{\sqrt{3}\,k\alpha_1\left[\left(\frac{k\alpha_1}{3}\right)^2+\frac{\alpha_1}{2}-\xi_1^2\right]}{\left[\frac{(k\alpha_1)^2}{3}+\xi_1^2+\alpha_1\right]^{\frac{3}{2}}}.$$

The only negative root here is ω_3.

2. **Identical capillaries** ($\alpha_1 = \alpha_2 = \alpha$):

$$\omega_{1,2} = -\omega_{3,4} = \left\{\frac{\alpha^2 k + 2\alpha + \xi_1^2}{2} \pm \left[\left(\frac{\alpha^2 k^2 - \xi_1^2}{2}\right)^2 + \right.\right.$$
$$\left.\left. + \alpha\left(\alpha^2 k^2 + \xi_1^2\right) + \alpha^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}. \tag{5}$$

It ensues from the previous computations that the radicand is always nonnegative.

The following thermal conditions must be fulfilled at the heated surface:

$$-\lambda_z \frac{dT^*}{dz} = q_0^*, \quad T_1^* = T_2^*. \tag{6}$$

T^*, T_1^*, and T_2^* tend to zero at infinity.

A solution for the system (2), meeting the conditions (6), is derived from the general solution via truncating the terms to positive powers $\exp(\omega_i z)$ and converting to the originals:

$$T(z, r) = \int_0^\infty q_0^* \frac{J_0(\xi r)\xi}{\lambda_z \xi_1^2(\omega_3^2 - \omega_4^2)}\left[(\omega_4^2 - \xi_1^2)\omega_3 \times\right.$$
$$\left. \times \exp(\omega_3 z) - (\omega_3^2 - \xi_1^2)\omega_4 \exp(\omega_4 z)\right] d\xi,$$

$$T_1(z, r) = \int_0^\infty q_0^* \frac{J_0(\xi r)\xi\alpha_1 k}{\lambda_z \xi_1^2(\omega_3^2 - \omega_4^2)}\left[\frac{\omega_4^2 - \xi_1^2}{\alpha_1 k - \omega_3}\omega_3 \times \exp(\omega_3 z) - \frac{\omega_3^2 - \xi_1^2}{\alpha_1 k - \omega_4}\omega_4 \exp(\omega_4 z)\right] d\xi, \tag{7}$$

$$T_2(z, r) - T_1(z, r) = \int_0^\infty q_0^* \frac{J_0(\xi r)\xi k}{\lambda_z \xi_1^2(\omega_3^2 - \omega_4^2)} \times [\exp(\omega_3 z) - \exp(\omega_4 z)]\,(\omega_4^2 - \xi_1^2)(\omega_3^2 - \xi_1^2)\, d\xi.$$

If the heat flux is distributed over the surface by the Gaussian law

$$q_0(r) = q_0 \exp\left(-\frac{r^2}{r_0^2}\right), \tag{8}$$

then the value of the function q_0^* in Eqs. (7) is

$$q_0^* = q_0 \frac{r_0^2}{2} \exp\left(-\xi^2 \frac{r_0^2}{4}\right), \tag{9}$$

whereas, with a uniform distribution in the spot of radius r_0, it is

$$q_0^* = q_0 \frac{r_0}{\xi} J_1(\xi r_0). \tag{10}$$

When the offtakes are thermally insulated, the solution simplifies:

$$T(z, r) = -\int_0^\infty q_0^* \frac{J_0(\xi r)\xi}{\lambda_z \omega_3} \exp(\omega_3 z)\, d\xi. \tag{11}$$

The stress–strain state is identified for an isotropic half-space by elastic properties, because the allowance for constructive anisotropy, while causing no fundamental difficulties, leads to very cumbersome expressions, thereby bringing a mathematical model not much closer to the real cooling system.

The thermoelastic problem is formulated thus. In an isotropic half-space $z \geq 0$, with a surface free from mechanical loads, it is required that elastic stresses and displacements be determined, which originate under the effect of the above-found temperature field and decay at infinity.

The solution to the thermoelastic problem can be represented in terms of the Goudier thermoelastic potential φ and of two Boussinesq harmonic functions f and g [4]:

$$
\begin{aligned}
u_r &= \frac{\partial(\Phi+\varphi)}{\partial r}, \\
u_z &= \frac{\partial(\Phi+\varphi)}{\partial z} 4(1-\nu) f, \\
\sigma_{rr} &= 2\mu\left(\frac{\partial^2\Phi}{\partial r^2} - \nu\nabla^2\Phi + \frac{\partial^2\varphi}{\partial r^2} - \nabla^2\varphi\right), \\
\sigma_{\varphi\varphi} &= 2\mu\left(\frac{1}{r}\frac{\partial\Phi}{\partial r} - \nu\nabla^2\Phi - \nabla^2\varphi + \frac{\partial\varphi}{\partial r}\frac{1}{r}\right), \\
\sigma_{zz} &= 2\mu\left[\frac{\partial^2\Phi}{\partial z^2} - (2-\nu)\nabla^2\Phi + \frac{\partial^2\varphi}{\partial z^2} - \nabla^2\varphi\right], \\
\sigma_{rz} &= 2\mu\left[\frac{\partial^2\Phi}{\partial r\partial z} - 2(1-\nu)\frac{\partial f}{\partial r} + \frac{\partial^2\varphi}{\partial r\partial z}\right],
\end{aligned} \tag{12}
$$

where $\Phi = fz + g$.

Forms of the harmonic functions and thermoelastic potential are determined from the boundary conditions and from the above-obtained temperature field in the cooling system frame, since the thermoelastic potential should satisfy the equation

$$\nabla^2\varphi = mT. \tag{13}$$

Let us now convert, using the Hankel transform, to representations of the functions entering into Eq. (12)

$$
\begin{aligned}
\begin{Bmatrix} u_z^* \\ \sigma_{zz}^* \end{Bmatrix} &= \int_0^\infty \begin{Bmatrix} u_z(z, r) \\ \sigma_{zz}(z, r) \end{Bmatrix} J_0(\xi r)\, r\, dr, \\
\begin{Bmatrix} u_r^* \\ \sigma_{rz}^* \end{Bmatrix} &= \int_0^\infty \int_0^r \begin{Bmatrix} u_r(z, \rho) \\ \sigma_{rz}(z, \rho) \end{Bmatrix} d\rho\, J_0(\xi r)\, r\, dr.
\end{aligned} \tag{14}
$$

Then, the solution of the thermoelastic problems in images, which complies with the boundary conditions

$$\sigma_{rz}(0, r) = \sigma_{zz}(0, r) = 0$$

and damps at infinity, appears as

$$
\begin{aligned}
u_z^* &= -(-1 + 2\nu - \xi z)\exp(-\xi z) F + F_1, \\
u_r^* &= -\left[z - \frac{2(1-\nu)}{\xi}\right]\exp(-\xi z) F + F_2, \\
\sigma_{zz}^* &= -2\mu\xi^2 z \exp(-\xi z) F + 2\mu\xi^2 F_2, \\
\sigma_{rz}^* &= -2\mu(1 - z\xi)\exp(-\xi z) F + 2\mu F_1,
\end{aligned} \tag{15}
$$

where

$$
\begin{aligned}
F_2 &= \frac{q_0^* m}{\lambda_z(\omega_3^2 - \omega_4^2)\xi_1^2}\left\{\frac{\omega_4(\omega_3^2 - \xi_1^2)}{\omega_4^2 - \xi^2}[\exp(-\xi z) - \right. \\
&\left. - \exp(\omega_4 z)] - \frac{\omega_3(\omega_4^2 - \xi_1^2)}{\omega_3^2 - \xi^2}[\exp(-\xi z) - \exp(\omega_3 z)]\right\}; \\
F &= -q_0^* \frac{m[\omega_3\omega_4(\omega_3 + \omega_4 - \xi) - \xi\xi_1^2]}{\lambda_z(\omega_3 + \omega_4)(\omega_3 - \xi)(\omega_4 - \xi)\xi_1^2}; \\
F_1 &= \frac{dF_2}{dz}.
\end{aligned}
$$

Of greatest interest are the stresses and thermal displacements directly at the heated surface. Obviously $F_2(0) = 0$. After performing simple manipulations we can make certain that $F_1(0) = F$. Therefore, at the surface we obtain

$$
\begin{gathered}
u_z^* = 2(1-\nu)F, \\
u_r^* = 2\,\frac{1-\nu}{\xi}\,F, \\
T^* = \frac{q_0^*}{\lambda_z \xi_1^2 (\omega_3^2 - \omega_4^2)}\,[(\omega_4^2 - \xi_1^2)\,\omega_3 - (\omega_3^2 - \xi_1^2)\,\omega_4].
\end{gathered}
\tag{16}
$$

Using the inverse Hankel transform, we pass to the original solution:

$$
\begin{gathered}
\begin{Bmatrix} u_z(z,\ r) \\ \sigma_{zz}(z,\ r) \end{Bmatrix} = \int_0^\infty \begin{Bmatrix} u_z^* \\ \sigma_{zz}^* \end{Bmatrix} J_0(\xi r)\,\xi d\xi, \\
\begin{Bmatrix} u_r(z,\ r) \\ \sigma_{rz}(z,\ r) \end{Bmatrix} = -\int_0^\infty \begin{Bmatrix} u_z^* \\ \sigma_{rz}^* \end{Bmatrix} J_1(\xi r)\,\xi^2 d\xi, \\
\sigma_{rr}(z,\ r) = \int_0^\infty \Big\{ 2\mu \Big[-\frac{\xi^2}{1-\nu} J_0(\xi r) + \frac{\xi}{r} J_1(\xi r) \Big] u_r^* + \\
+ \Big(\frac{\nu}{1-\nu}\,\sigma_{zz}^* - 2\mu m T^* \Big) J_0(\xi r) \Big\}\,\xi d\xi, \\
\sigma_{\varphi\varphi}(z,\ r) = \int_0^\infty \Big\{ 2\mu \Big[-\frac{\nu\xi^2}{1-\nu} J_0(\xi r) - \frac{\xi}{r} J_1(\xi r) \Big] u_r^* + \\
+ \Big(\frac{\nu}{1-\nu}\,\sigma_{zz}^* - 2\mu m T^* \Big) J_0(\xi r) \Big\}\,\xi d\xi.
\end{gathered}
\tag{17}
$$

The surface temperature is derived from Eqs. (7) and (11) by substituting $z = 0$. A profile of the surface deflections is then:

$$
u_z(0,\ r) = -\int_0^\infty \frac{q_0^* 2\beta (1+\nu)}{\lambda_z \xi_1^2}\,\frac{\omega_3\omega_4(\omega_3 + \omega_4 - \xi) - \xi\xi_1^2}{(\omega_3+\omega_4)(\omega_3-\xi)(\omega_4-\xi)}\,J_0(\xi r)\,\xi d\xi. \tag{18}
$$

Stresses in the center of the thermal loading spot are

$$
\begin{gathered}
\sigma_{rr}(0,\ 0) = \sigma_{\varphi\varphi}(0,\ 0) = \int_0^\infty \frac{2\mu q_0^* \beta (1+\nu)}{\lambda_z (1-\nu)\,\xi_1^2} \times \\
\times \Big\{ \frac{\omega_3\omega_4(\omega_3+\omega_4-\xi) - \xi\xi_1^2}{(\omega_3+\omega_4)(\omega_3-\xi)(\omega_4-\xi)}\,(1+\nu)\,\xi - \\
- \frac{(\omega_4^2 - \xi_1^2)\,\omega_3 - (\omega_3^2 - \xi_1^2)\,\omega_4}{\omega_3^2 - \omega_4^2} \Big\}\,\xi d\xi.
\end{gathered}
\tag{19}
$$

Thus, we have solved the thermoelasticity problem for the half-space, cooled by an opposing-reverse coolant flow and heated by a local heat flux at the surface.

Let us investigate the effect exerted by a coolant heating in a stress–strain state of the cooling system and, specifically, ascertain conditions in which it can be disregarded. The knowledge of these conditions will enable a significant simplification of the statement and solution of nonstationary thermoelasticity problems, since as long as the heating effect is small in a steady regime, it will be only smaller in a nonsteady regime and thus can be ignored.

Figure 2 plots dimensionless temperatures, deflections, and stress intensities in the center of a thermal loading spot with a Gaussian distribution

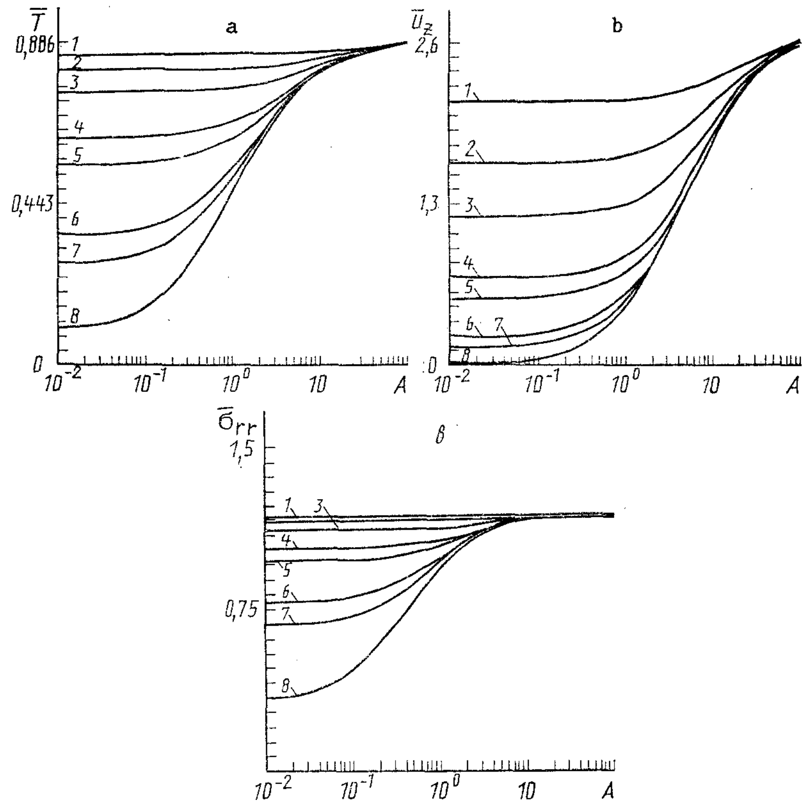

Fig. 2. Dimensionless temperatures (s), deflections (b), and stresses (c) of a heated surface as functions of the parameter A: 1) 0.01; 2) 0.032; 3) 0.1; 4) 0.5; 5) 1; 6) 5; 7) 10; 8) Bi = 100.

$$q(r) = q_0 \exp\left(-\frac{r^2}{r_0^2}\right)$$

in the cooling system with thermally insulated offtakes as functions of the dimensionless parameters

$$\mathrm{Bi} = \frac{\alpha_V r_0^2}{\lambda_r} \text{ and } A = \frac{k}{r_0}.$$

The first parameter characterizes the heat-transfer rate, whereas the second defines the effect of coolant heating. With rising A, the coolant heating and its influence on characteristics of the cooling system increase. For A > 10, the surface temperature is almost independent of Bi. A variation in the latter from 10^{-2} up to 10^2 causes a temperature change by not less than 10%. A relevant region for stresses is A > 2, and for deflections A > 20. As is clear from the figures, temperatures and stresses at the surface with A → ∞ are limited, while deflections in the center of the thermal loading spot grow monotonically.

Let the limiting values for A → ∞ be determined. From Eq. (3) we obtain the roots

$$\omega_3 = -\xi_1,\ \omega_4 = -\alpha_2 k.$$

The surface temperature and stresses in the spot center will be as follows:

$$T(0,\ 0) = \frac{q_0 r_0 \sqrt{\pi}}{2\sqrt{\lambda_z \lambda_r}},$$

$$\sigma_{rr}(0,\ 0) = -\frac{q_0 \mu r_0 \beta (1+\nu)\left(1 - \nu\sqrt{\frac{\lambda_z}{\lambda_r}}\right)\sqrt{\pi}}{\lambda_z (1-\nu)\left(1 + \sqrt{\frac{\lambda_r}{\lambda_z}}\right)}. \tag{20}$$

At $\lambda_r = \lambda_z$, Eqs. (20) coincide with the solution, acquired in [5], which has examined nonsteady thermal loading of an isotropic space with no cooling. It is interesting to point out that stresses are totally absent from the spot center when $\nu\sqrt{\lambda_z/\lambda_r} = 1$.

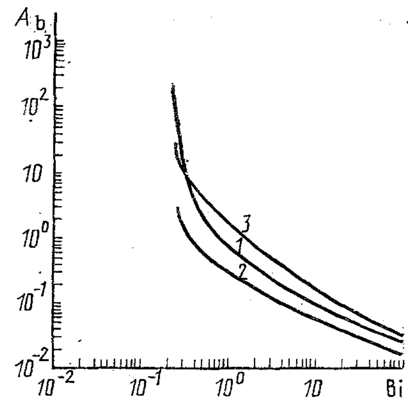

Fig. 3. Boundary values of the parameter A_b vs the Biot number: 1) by temperatures; 2) by displacements; 3) by stresses.

In the region of large A, heat transfer can be entirely neglected when calculating temperatures and stresses.

With small A, the temperature, deflections, and stresses cease to depend on the coolant heating, with a proper A region existing for each Bi. For example, ignoring the coolant heating at Bi = 0.5 results in an error of temperature computation not larger than 10%, provided A < 1.1. With Bi increasing up to 5, this region narrows down to A ≤ 0.12.

Consideration will be given to this limiting state. Equation (3) immediately yields the roots

$$\omega_3 = -\sqrt{\alpha_1 + \alpha_2 + \xi_1^2}\ , \quad \omega_4 = 0.$$

The temperature, deflections, and stresses in the center of the spot with a Gaussian distribution of heat flux density are the following

$$T(0,\ 0) = \frac{q_0 r_0 \sqrt{\pi}}{2\sqrt{\lambda_z \lambda_r}} \exp\left(\frac{r_0^2(\alpha_1+\alpha_2)\lambda_z}{4\lambda_r}\right) \operatorname{erfc} \sqrt{\frac{r_0^2(\alpha_1+\alpha_2)\lambda_z}{4\lambda r}},$$

$$u_z(0,\ 0) = -\int_0^\infty \frac{q_0 r_0^2 \beta (1+\nu) \exp\left(\frac{\xi^2 r_0^2}{4}\right) \xi d\xi}{\lambda_z (1-\nu)\sqrt{\alpha_1+\alpha_2+\xi_1^2}\ (\sqrt{\alpha_1+\alpha_2+\xi_1^2}+\xi)}, \tag{21}$$

$$\sigma_{rr}(0,\ 0) = -\int_0^\infty \frac{q_0 r_0^2 \mu \beta (1+\nu) \exp\left(-\frac{\xi^2 r_0^2}{4}\right)(-\nu\xi + \sqrt{\alpha_1+\alpha_2+\xi_1^2})\xi d\xi}{\lambda_z (1-\nu)\sqrt{\alpha_1+\alpha_2+\xi_1^2}\ (\sqrt{\alpha_1+\alpha_2+\xi_1^2}+\xi)}.$$

It can be inferred by comparing Eqs. (20) and (21) that temperature values for Bi = 0.03 differ by no more than 10%. Hence, heat transfer in this region can also be disregarded. In computing stresses, this region expands to Bi ≤ 0.03.

Thus, the conditions when not only heating but heat transfer can be neglected altogether in calculating temperatures and stresses on a heated surface of the capillary cooling system, are Bi ≤ 0.03 or A > 10 in the first case and Bi ≤ 0.03 or A ≥ 2 in the second.

Figure 3 gives boundary values A_L vs the parameter Bi. If $A \le A_b$ in the cooling system, then the surface temperatures and stresses can be predicted accurately up to 10%, taking no account of the coolant heating.

If the indicated conditions are not fulfilled, then for determining the stress-strain state of the capillary cooling systems the above solution (7), (15)-(19) must be used.

NOTATION

α_{v1}, α_{v2}, volume coefficients of heat transfer for the coolant supply and removal; ρ, c, v, density, specific heat, and filtration rate of the coolant; λ_r, λ_z, thermal conductivities of the framework; μ, β, ν, displacement modulus, coefficient of linear expansion, and Poisson coefficient of the framework; u_r, u_z, components of heat transference; σ_{rr}, $\sigma_{\varphi\varphi}$, σ_{rz}, σ_{zz}, components of stress tensor; q, normal component of heat flux; T, T_1, T_2, temperature of the framework and of the supplied and removed coolant; q_0, r_0, intensity and rate of thermal load with the Gaussian distribution; ξ, parameter of the Hankel transform;

$$\xi_1 = \xi \frac{\lambda_r}{\lambda_z};\quad \Delta T = T_2 - T_1;\quad \alpha_i = \frac{\alpha_{vi}}{\lambda_z};\quad \mathrm{Bi} = \frac{(\alpha_{v1} + \alpha_{v2})\, r_0^2}{\lambda_r};\quad k = \frac{\lambda_z}{\rho c v};\quad A = \frac{k}{r_0};$$

$$m = \beta \frac{1+\nu}{1-\nu};\quad \nabla_r^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r};\quad \overline{T} = \frac{T\lambda_z}{q_0 r_0};\quad \overline{\sigma} = \frac{\sigma\lambda_z}{q_0 r_0^2 \beta\mu};\quad \overline{u} = \frac{u_z \lambda_z}{q_0 r_0^2 \beta}.$$

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